ESTIMATION OF PHASE BOUNDARY BY FRONT POINTS METHOD IN ELECTRICAL IMPEDANCE TOMOGRAPHY

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ABSTRACT

Numerical works are conducted to develop a visualization technique for the phase distribution in a two-phase system by electrical impedance tomography technique, which reconstructs the conductivity distribution with the electrical responses that are determined by corresponding excitations. For the two-phase flow system, the impedance of each phase can be known but instead the phase boundary depending on the distribution of dispersed phase is of interest. In the present study, the image reconstruction problem is derived as a boundary estimation one and a new algorithm is developed for the estimation of phase boundary based on the front points tracking technique. To test the robustness of the proposed algorithm some numerical simulations are conducted. Numerical works show that the proposed algorithm can treat two-phase systems reasonably even with some errors in measurement data.

INTRODUCTION

Two-phase flow can occur under the normal and accidental conditions in various processes such as heat exchanger, steam power generation and oil or natural gas pumping system. Because the heterogeneous phase affects the safety, control, operation and optimization of process, it is important to know the phase boundaries in on-line without disturbing the flow field. Recently, the electrical tomography technique is employed to investigate two-phase flow phenomena, because it is relatively inexpensive and has good time resolution. In the present study, an electrical impedance tomography (EIT) measurement system and an image reconstruction algorithm were developed for the detection and visualization of multi-phase flow.

Since it is well-known that the conventional regularized Newton-Raphson can hardly identify the phase boundary even for the two-phase systems [1, 2], some novel attempts to reconstruct two-phase image were made on the basis of mesh-grouping [3, 4], adaptive mesh regeneration [5], and adaptive mesh refinement algorithms [6]. The key idea of the mesh grouping resorts to the fact that two-phase flow has only two conductivity values. After ordinary iteration procedures, meshes having similar conductivity values and progresses are classified into three groups, namely base group (e.g. liquid), object group (e.g. vapor) and unadjusted group, then each of the base group and the object group is thought of as a single mesh with a corresponding unknown conductivity. As a consequence, the number of unknowns can be reduced and the sensitivity to the conductivity change is magnified. In the adaptive mesh regeneration, to avoid the blurred image reconstructed by conventional algorithms the FEM mesh structure is fitted adaptively to the real phase boundaries. In the adaptive mesh refinement, an emphasis is placed on the increase of the computational efficiency when the meshes are refined to obtain the EIT images with higher spatial resolution. Molinari et al. [7] developed an algorithm which automatically produced finer meshes in areas where there were sharp gradients of conductivity distribution in the EIT images based on an *a posteriori* error estimate in order to adapt the mesh structure to phase boundaries.

Recently, boundary estimation technoque [8] are introduced for the two-phase flow system where the impedance of each phase doesn't change but instead the phase boundary depends on the distribution of dispersed phase. In this method, the boundary shape and loaction are estimated rather than the conductivities of each pahse.

In the present study, a new reconstruction algorithm for electric impedance imaging technique is introduced to visualization of phase distribution in two-phase systems. The phase boundaries are expressed as the discrete front points points located discretely along the boundary rather than Fourier series which was commonly used in most previous works for the boundary estimation and the front points are tracked with the aid of inverse problem algorithm in the context of finite element calculation.

MATHEMATICAL MODEL

Definition of the Problem

Let $\Omega \subset \Re^2$ denote a bounded domain and assume that Ω is divided into $m+1$ disjoint regions S_k which are bounded by smooth closed boundary curves and have constant conductivity values $\{\sigma_k, k = 0,1,\cdots,m\}$.

$$\Omega = \bigcup_{k=0}^{m} S_k \ , \quad k = 0,\cdots,m \qquad (1)$$

Let $\{C_\ell\} \subset \Omega$, $\ell = 1,2,\cdots,m$ denote the smooth boundary of region S_k . Assume that $\partial\Omega$ a missing. In this case it is interesting to find the
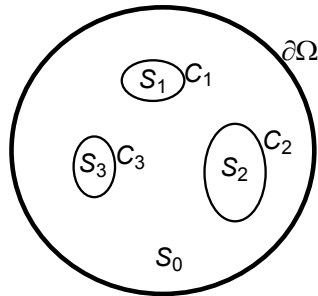
region nd the values $\{\sigma_k\}$ are known *a* priori but some details of the geometrical information on the boundaries $\{C_\ell\}$ are boundaries $\{C_\ell\}$ with the known conductivity values and the known boundary current inputs and potentials. This situation occurs in the multiphase flow visualization problem where the boundary of system and the physical properties such as electrical conductivity are known *a priori* but the phase boundaries are to be determined. Figure 1 shows the topology of the problem domain.

In the present study, a novel algorithm to estimate the phase boundaries based on the EIT technique will be proposed. In the proposed method, the shape and location of boundaries $\{C_\ell\}$ are approximated as an interpolation with front points located discretely along the boundary instead of Fourier series which was used in most previous works for the boundary estimation [5, 8] and the aid of forward problem of EIT which is based on the finite element method (FEM) discretization. The inverse problem is to find , the shape and location of boundaries $\{C_\ell\}$ with the applied input current and the measured boundary voltages. The details of the forward and the inverse problems are discussed below.

Forward Problem

When electrical currents $I_\ell (\ell = 1,2,\cdots,L)$ are injected into the object $\Omega \in \Re^2$ through the electrodes $e_\ell (\ell = 1, 2,\cdots, L)$ attached on the boundary $\partial\Omega$ and the conductivity distribution is known over the Ω, the corresponding electrical potential $u(\mathbf{x})$ on the Ω can be determined uniquely from the partial differential equation, which can be derived from the Maxwell's equations, and the Neuman type boundary conditions

$$\nabla \cdot (\sigma \nabla u) = 0 , \ \mathbf{x} \in \Omega \qquad (2)$$

$$\int_{e_\ell} \sigma \frac{\partial u}{\partial \nu} dS = I_\ell , \ \mathbf{x} \in e_\ell , \ \ell = 1,2,\cdots,L \qquad (3)$$

$$\sigma \frac{\partial u}{\partial \nu} = 0 , \ \mathbf{x} \in \partial\Omega \Big/ \cup_{\ell=1}^{L} e_\ell \qquad (4)$$

where e_ℓ is the ℓ'th electrode, ν is the outward directed unit normal vector and L is the number

Figure 1. Example of a piecewise constant domain.

of electrodes. Various forms of the boundary conditions to treat electrodes have been derived, and we employ the complete electrode model (CEM) which takes into account the discrete electrodes, the effects of the contact impedance, and the shunting effect of the electrodes. In the CEM, the boundary voltages on the electrodes are obtained as:

$$u + z_\ell \sigma \frac{\partial u}{\partial \nu} = U_\ell, \quad \mathbf{x} \in e_\ell, \quad \ell = 1, 2, \cdots, L \qquad (5)$$

where z_ℓ is the effective contact impedance between the ℓ'th electrode and the object, and U_ℓ is the voltage on the ℓ'th electrode. In addition, the following two conditions for the injected current and measured voltages are needed to ensure the uniqueness of the solution.

$$\sum_{\ell=1}^{L} I_\ell = 0 \qquad (6a)$$

$$\sum_{\ell=1}^{L} U_\ell = 0 \qquad (6b)$$

In general, the forward problem cannot be solved analytically, thus we have to resort to the numerical method. There are different numerical methods such as the finite difference method (FDM), boundary element method (BEM), and finite element method (FEM). In this study, we use the FEM to obtain numerical solution. In the
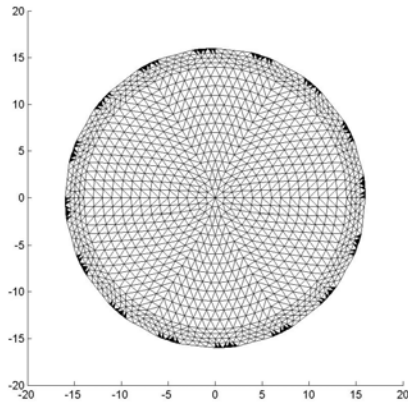
FEM, the object area is discretized into small elements having a node at each corner, as shown in Figure 2. And, with the aid of Figure 3, the effective conductivity value σ_e of element e which is intercepted by the region boundary C_ℓ can be expressed as [9]

$$\sigma_e = \frac{\sigma_l S_l + \sigma_r S_r}{S_e (= S_l + S_r)}, \qquad (7)$$

where S denotes the area. The value of σ_l is the conductivity in the region S_l, which is closed by the curve C_ℓ, and σ_r is the conductivity in the surrounding region S_r, as shown in Figure 3. So, the resistivity distribution within an element can be assumed to be constant and the standard FEM can be used to calculate the boundary potential. By employing FEM, the potential at each node is calculated by discretizing Eq. (1) into $Yv = c$, where $Y \in \Re^{N \times N}$ is the admittance matrix that is a function of resistivity and c represents the current injected into the object. The detailed descriptions on the forward problem are given in Vauhkonen's work [10].

Inverse Problem

The inverse problem of EIT maps the boundary voltages to a conductivity distribution, which is obtained by minimizing the following object functional
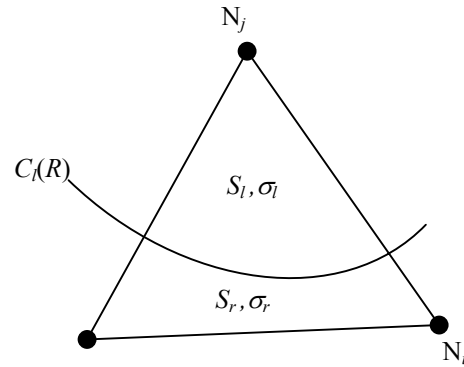

Figure 2. FEM mesh used in the present study. Locations of the electrodes are marked with darkened elements.

Figure 3. A schematic representation of FEM element Ω_e intercepted by the phase boundary $C_\ell(R)$.

$$\Phi = \frac{1}{2}\left[(V-U)\right]^T\left[(V-U)\right]. \tag{8}$$

where V is the vector of the measured voltages and U is the vector of the calculated boundary voltage. In conventional EIT problem, the domain is discretized into small elements, in each of which the conductivity is assumed to be constant. The inverse problem of EIT is to find the conductivity value of each small element. Because of the mismatch between the FEM mesh structure and the real phase distribution, the blurred image is unavoidable even though the system is composed of only two phases and the error-free synthetic data is used as the measured voltage vector. In fact, our major concern in the application of EIT to visualization of two-phase flows is to reconstruct the phase boundaries rather than the conductivity value of each phase. So, we reformulate the EIT inverse problem into a boundary estimation one. If we choose M points spacing equivalently along the polar angle in the frame of polar coordinate, the angle position will be

$$\theta_k = 2\pi(k-1)/M, \quad k = 1,2,\cdots,M. \tag{8}$$

The set of front points, R, is defined as follows:

$$R = \{r_k \mid k = 1,2,\cdots,M\} \tag{9}$$

where r_k is the distance of the k-th front point measured from the fixed center of the reference coordinate (0,0). To express the boundary with the discrete front points, an interpolation is
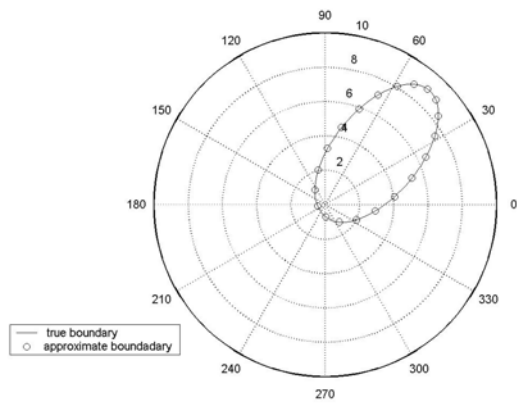

Figure 4. Phase boundary approximated by front points.

required. In this paper Fourier interpolation is used. Figure 4 shows an example of boundary approximation with front-point concept.

The mapping F from the coefficients r_k's to the measured potential is highly nonlinear. We linearize the mapping $F : R_k \rightarrow U$ at a certain point R_k^* to obtain

$$U = U_* + \mathbf{J}_F\left(R_k - R_k^*\right) \tag{10}$$

where U_* are the calculated potentials corresponding to R_k^* and $\mathbf{J}_F = \partial U/\partial R$ is the Jacobian matrix. It is well-known that the inverse EIT problem suffers from ill-posedness and this ill-posedness can be mitigated by the regularization [1,10]. Then, the iterative equation for the incremental change of R_k is obtained by using Levenverg –Marquardt regularization as

$$\Delta R = \left(\mathbf{J}_F^T\mathbf{J}_F + \kappa\mathbf{I}\right)^{-1}\left(\mathbf{J}_F^T(V-U_*)\right) \tag{11}$$

where V is the measured voltages. There are several methods for choosing in some sense optimal regularization parameters κ. However, the different criteria will yield results of different optimality. Since the true distributions in the present study are known, the regularization parameter is chosen to obtain the best reconstructed images. So, the regularization parameter is set to $\kappa = 0.05$.

In every iteration of inverse problem, we have to solve the forward problem in order to obtain the boundary voltages and the Jacobian matrix. There are several methods to calculate the Jacobinan matrix in standard EIT problems, where Jacobian means the change of boundary voltages with respect to the change of the resisitivity distribution, that is $J(\sigma_e) = \partial U/\partial \sigma_e$. However, in this study, the standard perturbation method has been employed to calculate the Jacobian matrix, $J_F(R)$, since the analytic method are not well-established. The column of $J_F^{r_m}$ of $\mathbf{J}_F$ are obtained by perturbing each of the coefficient R_k by predetermined small δR_k and calculating the changes of resulting voltages on the boundary.

$$J_F(R_k) = \frac{\delta U}{\delta R_k}, \qquad (12)$$

where δU is the resulting voltage difference vector.

NUMERICAL SIMULATIONS

Unlike many other types of imaging techniques, making general statements about the limitations of imaging with EIT is difficult. The resolution of the EIT system depends on the various variables, such as conductivity contrast and distribution, injected current pattern, and the errors in current injection and voltage measurement. Therefore, to verify the appropriateness of the present EIT system, a series of simulations should be conducted.

We consider a circular object of radius 14cm, which has 16 electrodes along the boundary. The domain is discretized into 1968 triangular elements in the finite element calculation. The resistivity values of the anomaly and the background are set to 600Ωcm and 300Ωcm, respectively. There are many data collecting methods, such as neighboring method, cross method, opposite method, multi-reference method and adaptive method. The characteristic of these methods are summarized in Webster's book [11]. Among these, the multi-reference method, where desired current distribution can be obtained by injecting current through all the electrodes simultaneously, is known to be the best one when there is no prior information on the conductivity distribution. In the present study, currents generated by current generation circuit are injected into the 16 electrodes simultaneously in the following form

$$I_\ell^k = \begin{cases} \cos(k\zeta_\ell) & k = 1,2,\cdots,8 \\ \sin(k\zeta_\ell) & k = 1,2,\cdots,7 \end{cases} \qquad (13)$$

where $\ell = 1,2,\cdots,16$ and $\zeta_\ell = \ell/16$, and the resulting potentials are measured simultaneously

To investigate the effect of the conductivity distribution and the measurement error level on the resolution of reconstructed image, we consider several artificial conductivity distributions and obtain the synthetic boundary voltages by using the forward solver described earlier. The error is inevitable in reality of EIT applications and it will loosen the relationship between the injected current patterns and the measured boundary voltages. To test the robustness of our algorithm against the measurement error and to avoid inverse crime, image reconstruction of simulated two-phase systems is conducted with assuming uniformly distributed random error of 1% in voltage measurement.

In Figure 5, we consider an artificial single object located at the center. Although it seems to be quite simple, this is a very illustrative example since the conductivity chage of the centered target
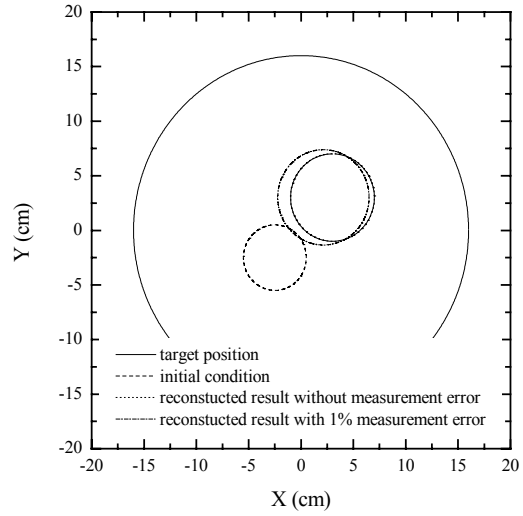

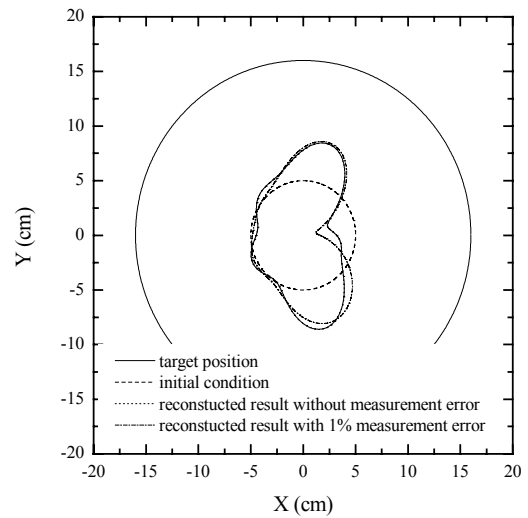Figure 5. Numerical simulation results for the simple boundary shape.

Figure 6. Numerical simulation results for the complicated boundary shape.

is very insensitive to the boundary voltages. Also, we consider a target located off the center. As shown in this figure, the proposed method works well even for 1% of measurement error and is insensitive to the initial guess. Even if the mismatch between the assumed positions of object and reconstructed ones is intensified as the imposed measurement error increases, the size of the object is well reconstructed. Also, more complicated geometry is considered in Figure 5. As shown in this figure, only small portion of dispersed phase boundary is changed, and the proposed method reconstructs the size and location of dispersed phase, quite well. If we formulate a boundary estimation problem for the above example based on Fourier series, higher mode Fourier coefficients (more than tenth) are required [12]. If the boundary is deformed in part as in the case of the example, the Fourier coefficients should be altered totally. However, in the proposed algorithm, only a few front points need to be modified. This is one of major advantages of this method over the previous methods, such as boundary estimation based on Fourier series.

The root mean square error (RMSE) defined as

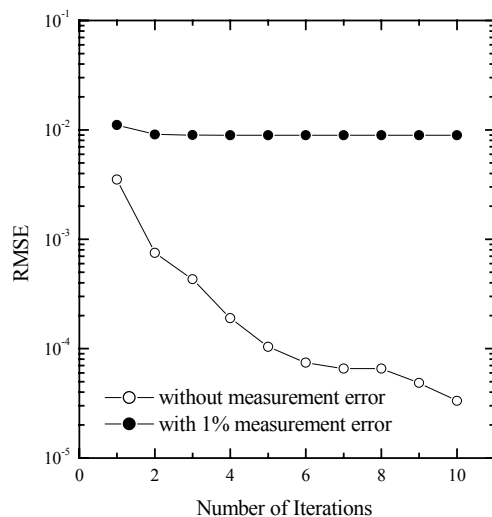$$\varepsilon = \sqrt{\frac{(V-U)^T(V-U)}{U^T U}} \tag{14}$$

is plotted for the one target system given in Figure 6. As shown in Figure 7, the RMSE decreases rapidly for the first two iteration, and shows nearly constant values after the third iteration. Other examples also exhibit the similar trends. Hence, the quality of reconstructed image doesn't improve after several iteration steps, and the maximum number of iterations is set to 10.

CONCLUSIONS

The present work intends to apply the EIT technique to the visualization of two-phase flow system. An algorithmic study is conducted to estimate the phase boundary by EIT. To resolve the problem that the conventional EIT can hardly detect the interfacial boundaries due to its inherent diffusive characteristic in the inverse estimation of conductivity distribution, resulting in the blurredness in reconstructed images, we reformulate the EIT inverse problem as the boundary estimation problem. The boundary is approximated as an interpolation of front points rather than Fourier coefficients and the position of the front points are tracked with the Newton-Raphson method. The reconstructed images based on our boundary estimation algorithm show that the phase boundary can be identified by the proposed method. It is expected that the EIT can be used to monitor various process systems where the two-phase and/or two-component transport exists.

ACKNOWLEDGEMENTS

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Figure 7. RMSE values for the case of Figure 6.

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